# Algebraic Approach to Ridge-Regularized Mean Squared Error Minimization in Minimal ReLU Neural Network

Akifumi Okuno
*The Institute of Statistical Mathematics*

**Abstract**

This paper investigates a perceptron, a simple neural network model, with ReLU activation and a ridge-regularized mean squared error (RR-MSE). Our approach leverages the fact that the RR-MSE for ReLU perceptron is piecewise polynomial, enabling a systematic analysis using tools from computational algebra. In particular, we develop a Divide-Enumerate-Merge strategy that exhaustively enumerates all local minima of the RR-MSE. By virtue of the algebraic formulation, our approach can identify not only the typical zero-dimensional minima (i.e., isolated points) obtained by numerical optimization, but also higher-dimensional minima (i.e., connected sets such as curves, surfaces, or hypersurfaces). Although computational algebraic methods are computationally very intensive for perceptrons of practical size, as a proof of concept, we apply the proposed approach in practice to minimal perceptrons with a few hidden units.